Author's Reply

J. C. Monzon

I wish to thank Volakis and Anastassiu for their interest shown in the above comments. Before directly responding to their comments, I would like to point out that I have recently been exposed[2] to a related work by Fel'd [1]. One of the equations, namely (24) in the above paper, was derived by Fel'd by alternative means. Neither the reviewers, myself, nor Volakis and Anastassiu knew of this reference, perhaps because the *Soviet Physics Doklady* is not very accessible to American engineers, and also because of its rather unusual title: "A quadratic lemma of electrodynamics." I believe the work of Fel'd deserves recognition in this TRANSACTIONS. However, I would like to state that my work encompasses the above paper and its generalization to more complex materials [2], and was done in 1991 (under Naval Research Laboratory (NRL) sponsorship), i.e., a year earlier than the paper by Fel'd.

Volakis and Anastassiu point out two things: 1) that (26) of the above paper has a factor of 1/2 in error; and 2) that (28) of the above paper can be derived easily by alternative means.

With regard to the factor 1/2, I believe it should be there since (26) is used to augment (24) in the sense that it is added on both sides. This is done appropriately by switching indices so as to present (24) with a statement of reciprocity in the usual operator sense, i.e.,

$$\mathrm{O}_+(1,\,2) = \mathrm{O}_+(2,\,1) \tag{C1}$$

for

$$\mathrm{O}_+(1,\,2) = \int_V \mathrm{d}\tau[\overline{\mathrm{M}}^{(2)} \cdot \overline{\mathrm{E}}^{(1)} + \eta^2 \overline{J}^{(2)} \cdot \overline{\mathrm{H}}^{(1)}] + \frac{\eta^2}{2} \oint_S \mathrm{d}\overline{\mathrm{S}} \cdot \left[\overline{\mathrm{H}}^{(1)} \times \overline{\mathrm{H}}^{(2)} - \frac{1}{\eta^2} \overline{\mathrm{E}}^{(1)} \times \overline{\mathrm{E}}^{(2)}\right]. \tag{C2}$$

Similarly, (25) is added to each side of (23) appropriately resulting in the usual statement of reciprocity

$$\mathrm{O}_-(1,\,2) = \mathrm{O}_-(2,\,1) \tag{C3}$$

for

$$\mathrm{O}_-(1,\,2) = \int_V \mathrm{d}\tau[\overline{J}^{(1)} \cdot \overline{\mathrm{E}}^{(2)} - \overline{\mathrm{M}}^{(1)} \cdot \overline{\mathrm{H}}^{(2)}] + \frac{1}{2} \oint_S \mathrm{d}\overline{\mathrm{S}} \cdot [\overline{\mathrm{H}}^{(1)} \times \mathrm{E}^{(2)} + \overline{\mathrm{E}}^{(1)} \times \overline{\mathrm{H}}^{(2)}]. \tag{C4}$$

It should be noted that (25) and (26) were introduced in a casual manner because they were used in an argument just to show that (23) and (24) were independent. Equations (25) and (26) are never used on their own. It is for this reason that the infinite integrals in (23) and (24) are never reduced to integrals over volume V, such as I have done above.

With respect to the derivation of (24) in the paper, I do not think that the analysis is lengthy. The analysis leading to (24) is 1-1/2 TRANSACTIONS' pages long, where (24) is not the only significant result, but also important is (19), the statement of reciprocity of the characteristic modes. It should also be noted that the introduction of characteristic modes allows the results to be obtained in a natural fashion, wherein the new theorem appears like the natural complement of the accepted form (one being the sum of the u's, the other the difference).

The fact that (24) can be derived by alternative means is known to me; an anonymous reviewer was the first to point this out to me (see the acknowledgment in the paper). Once a final result is known, it can of course be re-derived in a variety of ways. For instance, the vector used by Volakis (in (1) of the comment) was not derived and has no justification other than to duplicate (24) of the paper. What Volakis and Anastassiu present here is essentially what the anonymous reviewer presented to me, and most importantly, follows the same steps of the paper by Fel'd.

To summarize, the factor of 1/2 is not in error, and the "shorter" method presented by Volakis and Anastassiu is already available in the Soviet literature.

Manuscript received August 12, 1996.

The author is with Lockeed Martin, Princeton, NJ 08540 USA.

Publisher Item Identifier S 0018-9480(97)00848-X.

[2] Thanks are extended to A. Lakhtakia for introducing the author to the work of Fel'd.

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[2] J. C. Monzon, "New reciprocity theorems for chiral, nonactive and biisotropic media," *IEEE Trans. Microwave Theory and Tech.,*. vol. 44, pp. 2299–2301, Dec. 1996.

Corrections to "Reconstruction of the Constitutive Parameters for an Ω Material in a Rectangular Waveguide"

Martin Norgren and Sailing He

I. THE DIRECT PROBLEM

Due to a mistake, certain parts of the analysis in the above paper[1] are incorrect. Here we present the necessary corrections. It is shown that the corrected formalism leads to improved reconstructions. We consider a homogeneous block of an Ω material, filling the region $0 \leq z \leq L$ in a metallic rectangular waveguide with cross section $0 \leq x \leq a$ and $0 \leq y \leq b$.

To repeat, from analysis of Maxwell's equations

$$\nabla \times \vec{\mathbf{E}} = -j\omega(\overline{\overline{\mu}}\vec{\mathbf{H}} + \overline{\overline{\zeta}}\vec{\mathbf{E}}) \qquad \nabla \times \vec{\mathbf{H}} = j\omega(\overline{\overline{\epsilon}}\vec{\mathbf{E}} + \overline{\overline{\xi}}\vec{\mathbf{H}}) \tag{1}$$

with a time and z dependence of $\exp(j\omega t - \gamma z)$, it can be shown [1] that TE_{m0} (and TE_{0n}) modes can exist. For the TE_{m0} modes propagating in the $+z$ direction, we have the following set of solutions

$$H_3^m = C_m \cos\left(\frac{m\pi x}{a}\right) \exp(-\gamma_m z) \tag{2}$$

Manuscript received June 8, 1996.

The authors are with the Department of Electromagnetic Theory, Royal Institute of Technology, Stockholm, S-100 44, Sweden.

Publisher Item Identifier S 0018-9480(97)00846-6.

[1] M. Norgren and S. He, *IEEE Trans. Microwave Theory Tech.,* vol. 43, no. 6, pp. 1315–1321, June 1995.

$$E_2^m = -\frac{j\omega a\mu_3}{m\pi} C_m \sin\left(\frac{m\pi x}{a}\right) \exp(-\gamma_m z) \tag{3}$$

$$H_1^m = \frac{a\mu_3(\gamma_m - \omega\sqrt{\epsilon_0\mu_0}\Omega)}{m\pi\mu_1} C_m \sin\left(\frac{m\pi x}{a}\right) \exp(-\gamma_m z) \tag{4}$$

$$E_1^m = H_2^m = E_3^m = 0 \tag{5}$$

where

$$\gamma_m = j\sqrt{\omega^2(\epsilon_2\mu_1 - \epsilon_0\mu_0\Omega^2) - \frac{\mu_1}{\mu_3}\left(\frac{m\pi}{a}\right)^2} \tag{6}$$

and where we take the square root in (6) with a nonnegative real part. Similarly, we can obtain another set of solutions which propagate in the $-z$ direction [note that γ_m should then be replaced with $-\gamma_m$ in (2)–(5)]. Combining the two sets of solutions, we can write the correct relations for the total tangential fields for TE_{m0} modes as follows:

$$\vec{\mathbf{E}}_m^\perp = [E_m^+ \exp(-\gamma_m z) + E_m^- \exp(\gamma_m z)] \sin\left(\frac{m\pi x}{a}\right)\mathbf{e}_y \tag{7}$$

$$\vec{\mathbf{H}}_m^\perp = \left[-\frac{E_m^+}{Z_m^+}\exp(-\gamma_m z) + \frac{E_m^-}{Z_m^-}\exp(\gamma_m z)\right] \sin\left(\frac{m\pi x}{a}\right)\mathbf{e}_x \tag{8}$$

($\mathbf{e}_x$, $\mathbf{e}_y$ are unit vectors in the x and y directions, respectively), where the constant amplitudes $E_m^\pm$ are to be determined by the boundary conditions at $z = 0,\ L$, and

$$Z_m^+ = \frac{j\omega\mu_1}{\gamma_m - \omega\sqrt{\epsilon_0\mu_0}\Omega} \tag{9}$$

$$Z_m^- = \frac{j\omega\mu_1}{\gamma_m + \omega\sqrt{\epsilon_0\mu_0}\Omega}. \tag{10}$$

Note that for the TE_{m0}-modes in the Ω material the transverse wave-impedance for a left-moving wave is different from the one for a right-moving wave cf. [(9) and (10)]. Thus, more information may be obtained if double-sided excitation is used (the mistake in the above paper[1] is that both impedances are taken to be Z_m^+, which is not correct).

First consider the case when the waveguide is excited from the left region $z < 0$. Then the tangential fields for the TE_{m0}-modes, in the vacuum regions, have the forms

$$\left.\begin{aligned}\vec{\mathbf{E}}_m^\perp &= E_m^i[\exp(-\gamma_{0m}z) + R_m^+\exp(\gamma_{0m}z)]\sin\left(\tfrac{m\pi x}{a}\right)\mathbf{e}_y\\ \vec{\mathbf{H}}_m^\perp &= \tfrac{E_m^i}{Z_{0m}}[-\exp(-\gamma_{0m}z) + R_m^+\exp(\gamma_{0m}z)]\sin\left(\tfrac{m\pi x}{a}\right)\mathbf{e}_x\end{aligned}\right\}, \quad z < 0 \tag{11}$$

$$\left.\begin{aligned}\vec{\mathbf{E}}_m^\perp &= T_m^+E_m^i\exp[-\gamma_{0m}(z-L)]\sin\left(\tfrac{m\pi x}{a}\right)\mathbf{e}_y\\ \vec{\mathbf{H}}_m^\perp &= -\tfrac{T_m^+E_m^i}{Z_{0m}}\exp[-\gamma_{0m}(z-L)]\sin\left(\tfrac{m\pi x}{a}\right)\mathbf{e}_x\end{aligned}\right\}, \quad z > L \tag{12}$$

where

$$\gamma_{0m} = j\sqrt{\omega^2\epsilon_0\mu_0 - \left(\frac{m\pi}{a}\right)^2}, \qquad Z_{0m} = \frac{j\omega\mu_0}{\gamma_{0m}} \tag{13}$$

and R_m^+, T_m^+ are the reflection and transmission coefficients, respectively, for the TE_{m0}-modes with the left-sided excitation. E_m^i is the amplitude of the incident field. From (7), (8), (11), (12), and continuity of the tangential fields at $z = 0,\ L$, we obtain

$$(1+R_m^+)(Z_m^+\mathrm{e}^{-\gamma_m L} + Z_m^-\mathrm{e}^{\gamma_m L}) - 2\frac{Z_m^+Z_m^-}{Z_{0m}}(1-R_m^+)\sinh\gamma_m L = (Z_m^+ + Z_m^-)T_m^+ \tag{14}$$

$$2(1+R_m^+)\sinh\gamma_m L - (1-R_m^+)\left(\frac{Z_m^+}{Z_{0m}}\mathrm{e}^{\gamma_m L} + \frac{Z_m^-}{Z_{0m}}\mathrm{e}^{-\gamma_m L}\right) = -\frac{Z_m^+ + Z_m^-}{Z_{0m}}T_m^+. \tag{15}$$

Similarly, for the right-sided excitation the corresponding reflection and transmission coefficients, denoted by R_m^- and T_m^-, respectively, satisfy

$$(1+R_m^-)(Z_m^-\mathrm{e}^{-\gamma_m L} + Z_m^+\mathrm{e}^{\gamma_m L}) - 2\frac{Z_m^+Z_m^-}{Z_{0m}}(1-R_m^-)\sinh\gamma_m L = (Z_m^+ + Z_m^-)T_m^- \tag{16}$$

$$2(1+R_m^-)\sinh\gamma_m L - (1-R_m^-)\left(\frac{Z_m^-}{Z_{0m}}\mathrm{e}^{\gamma_m L} + \frac{Z_m^+}{Z_{0m}}\mathrm{e}^{-\gamma_m L}\right) = -\frac{Z_m^+ + Z_m^-}{Z_{0m}}T_m^-. \tag{17}$$

From (14) to (17), we can uniquely determine the reflection coefficients $R_m^\pm$ and the transmission coefficients $T_m^\pm$ in a direct problem if the material parameters are known. We obtain

$$R_m^+ = \frac{\left[\frac{Z_m^+}{Z_{0m}} - 1\right]\left[\frac{Z_m^-}{Z_{0m}} + 1\right]\sinh\gamma_m L}{\left[\frac{Z_m^+Z_m^-}{Z_{0m}^2} + 1\right]\sinh\gamma_m L + \left(\frac{Z_m^+ + Z_m^-}{Z_{0m}}\right)\cosh\gamma_m L} \tag{18}$$

$$R_m^- = \frac{\left[\frac{Z_m^+}{Z_{0m}} + 1\right]\left[\frac{Z_m^-}{Z_{0m}} - 1\right]\sinh\gamma_m L}{\left[\frac{Z_m^+Z_m^-}{Z_{0m}^2} + 1\right]\sinh\gamma_m L + \left(\frac{Z_m^+ + Z_m^-}{Z_{0m}}\right)\cosh\gamma_m L} \tag{19}$$

$$\begin{aligned}T_m^+ &= T_m^-\\ &= \frac{\frac{Z_m^+ + Z_m^-}{Z_{m0}}}{\left[\frac{Z_m^+Z_m^-}{Z_{0m}^2} + 1\right]\sinh\gamma_m L + \left(\frac{Z_m^+ + Z_m^-}{Z_{0m}}\right)\cosh\gamma_m L}\\ &\equiv T_m.\end{aligned} \tag{20}$$

The last equation can be explained by the reciprocity of the Ω material.

II. The Inverse Problem

A. Determination of γ_m, Z_m^+, and Z_m^-

From (14) to (17) we obtain the following equation by eliminating Z_m^+/Z_{0m} and Z_m^-/Z_{0m}:

$$\cosh\gamma_m L = \frac{T_m^2 - R_m^+R_m^- + 1}{2T_m} \equiv a_m + jb_m \tag{21}$$

where a_m and b_m are real and measurable (hence the corresponding (38) in [1] is incorrect). Let

$$\gamma_m = \alpha_m + j\beta_m \tag{22}$$

where $\beta_m > 0$ [cf. the definition (6)], $\alpha_m \geq 0$ (since we only consider passive media). Thus

$$\cosh\alpha_m L\cos\beta_m L = a_m, \qquad \sinh\alpha_m L\sin\beta_m L = b_m \tag{23}$$

which gives α_m uniquely as [1]

$$\alpha_m = \frac{1}{L} \cdot \sinh^{-1}\sqrt{\frac{1}{2}[a_m^2 + b_m^2 - 1 + \sqrt{(a_m^2 + b_m^2 - 1)^2 + 4b_m^2}]}. \tag{24}$$

After α_m has been determined, we obtain

$$\beta_m = \tilde{\beta}_m + \frac{2p\pi}{L}, \qquad p = 0,\ 1,\ 2, \cdots \tag{25}$$

where $\tilde{\beta}_m$, $0 < \tilde{\beta}_m \leq 2\pi/L$, is uniquely determined from (23) and where the integer p must be determined from some additional information.

After the propagation constant γ_m has been determined from (23) for the TE_{m0} mode, we obtain the impedances $Z_m^\pm$ [cf. (15) and (17)]

$$Z_m^\pm = Z_{0m} \cdot \frac{2(1 - R_m^+ R_m^-)\sinh\gamma_m L \pm (R_m^+ - R_m^-)[2\cosh\gamma_m L - T_m]}{2(1 - R_m^+)(1 - R_m^-)\cosh\gamma_m L - T_m(2 - R_m^+ - R_m^-)}. \quad (26)$$

B. Modified Reconstruction of Ω and μ_1

Excite the waveguide from two sides (in sequence) with the TE_{10} mode at a frequency ω. From (9) and (10) it follows

$$\Omega = \frac{\gamma_m(Z_m^+ - Z_m^-)}{\omega\sqrt{\epsilon_0\mu_0}(Z_m^+ + Z_m^-)}, \quad m = 1 \quad (27)$$

$$\mu_1 = \frac{2\gamma_m Z_m^+ Z_m^-}{j\omega(Z_m^+ + Z_m^-)}, \quad m = 1 \quad (28)$$

(the parameters ϵ_1, ϵ_2, μ_2, and μ_3 can then be determined from excitation of the TE_{01} and TE_{0N} ($N \geq 2$) modes [1]).

C. Correct Reconstruction of ϵ_3

Since the remaining parameter ϵ_3 cannot be determined using TE_{m0} and TE_{0n} modes [1] (even with a repositioned sample), we have to use a free space measurement (on an Ω-slab) of the reflection and transmission for a normally incident plane wave (propagating in x direction) with the electric field polarized in z direction. The corresponding propagation constant

$$\gamma_f = j\omega\sqrt{\epsilon_3\mu_2} \quad (29)$$

can be determined in an analogous way as described in Section II-A [cf. (21)]. ϵ_3 is then obtained from the above equation (note that the formulas at the end of p. 1319 in the original paper are incorrect).

III. Numerical Results and Conclusions

Like in the original, the size of the Ω sample is $a = b = L = 0.1$ m and the frequency range is 3 $\sim$ 4.5 GHz. To test the stability of the reconstruction scheme, we have added 5% of random noises to both the real and imaginary parts of the reflection and transmission coefficients (note that in [1] only 3% noise has been used).

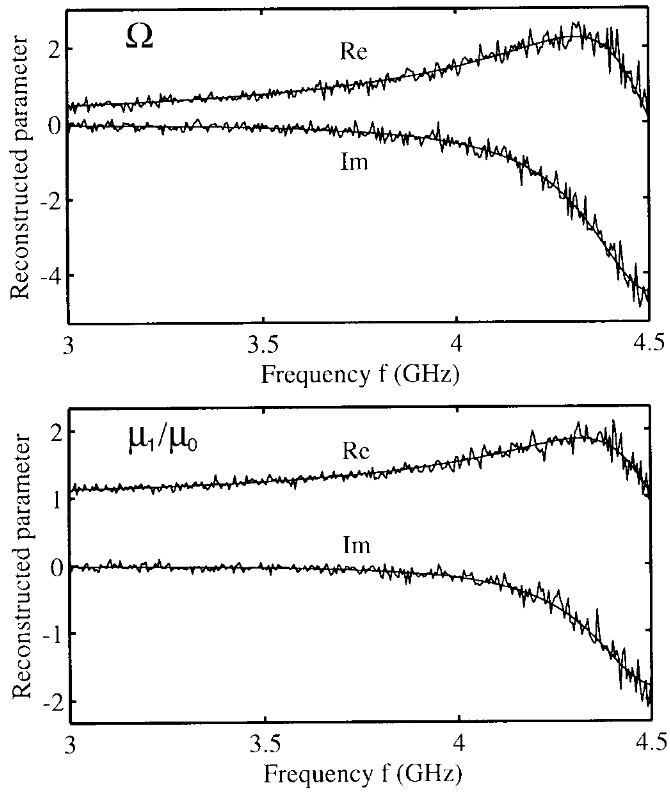

Fig. 1. The reconstruction of the dispersive parameters Ω and μ_1 using double-sided reflection and transmission data from the dominant TE_{10} mode.

The reconstruction of the parameters Ω and μ_1 using the noisy reflection and transmission data is showed in Fig. 1. Although the data is more noisy, the reconstruction seems to be better than in the above paper[1], which indicates that it is better to use the asymmetry of the sample together with double-sided excitation of the dominant TE_{10} mode instead of using single-sided excitation with additional higher order modes.